Event Triggered Adaptive Differential Modulation: A New Method for Traffic Reduction in Networked Control Systems

Upeka Premaratne, Saman K. Halgamuge and Iven M. Y. Mareels

***Abstract*—Congestion in a Networked Control System (NCS) has many undesirable effects that can make a control system unstable if severe enough. These include delays and packet drops. Therefore, reducing network traffic, including traffic generated by the NCS itself is a necessity. In Event Based Control (EBC) a control signal is generated when a specific event is triggered such as the control error exceeding a predetermined threshold. When the rate of change of a variable is bounded, event triggering reduces the effective frequency in which control signals have to be generated when compared to a system with periodic sampling. This paper proposes a new method of event triggering called Event Triggered Adaptive Differential Modulation (ETADM) that combines the bandwidth reduction strategies of event triggering and human speech coding techniques. The proposed method can be applied to nonlinear systems with a stabilizing feedback. In addition, it can be shown to be robust to packet drops. Stability for this method can be analyzed in terms of Input to State Stability (ISS) for a given bound of the signal reconstruction error.**

Keywords*—Networked control, event based control, input to state stability, adaptive differential modulation, signal reconstruction, event triggered adaptive differential modulation

I. Introduction

Event Based Control (EBC) is an emerging method of control where sampling and control signal generation takes place when a specific event is triggered instead of using a periodic clock signal. The triggering event could be a change in the quantum of control error (as determined by a prior set of quantization rules). It could also be the control variable exceeding a predetermined threshold such as the power output from a solar panel or wind turbine changing by a significant amount to require a change in the grid operation. When the rate of change of the input is bounded, the effective frequency of sampling and control signal generation is less when compared to periodic sampling. This is due to the reduced sampling during low variation and higher sampling during rapid changes caused by disturbances. This makes it suitable for embedded devices with limited computational resources as originally proposed by Arzen [1]. It is also favored for Network Control Systems (NCS) as shown in Fig. 1 where critical communication has to be minimized due to network congestion which in turn results in delays and packet dropouts. Studies in the recent past on NCS stability [2] [3] [4] [5] [6] [7], performance [8] [9] and effects of packet dropouts [10] [11] [12] [13] [14] have indicated the vulnerability and limitations of NCS to these issues.

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Out of the studies on the performance of EBC compared to periodic sampling, Arzen [1] compared both the tracking performance and computational cost. A comparison of performance with noise is given in [15]. A study by Aström and Bernhardsson [16] demonstrated that EBC can perform better than periodic sampling for large disturbances. Further confirmation of this result for stochastic problems can be found in [17].

Numerous event triggering mechanisms have been proposed for EBC. These include Memoryless Event Triggering (MET) [15] [18] where an event is triggered when the input variable exceeds a predetermined threshold. In Memory Based Event Triggering (MBET) [1] [19] [20], an event is triggered when the difference between the input and the stored input value during the previously triggered event exceed the threshold. In this method, a Zero Order Hold (ZOH) is needed to reconstruct the original signal. Prediction methods, where the next sampling time is predicted by the measured state and system dynamics have also been proposed [21] [22] [23].

A. Problem Statement

Traffic congestion in communication networks is a serious issue that can result in delays and packet drops. These can in turn, adversely affect the stability of a NCS. In a NCS it is possible to reduce traffic congestion by decreasing the effective sampling rate. EBC has been shown to be capable of decreasing the effective sampling rate [1] [16] [17] provided that the rate of change of the disturbance is bounded. Due to the numerous technically distinct methods used for event triggering, some promising methods such as MBET still lack a theoretically sound stability analysis. This makes them problematic to utilize for NCSs due to the difficulty to assess the impact of the delays and packet dropouts caused by the communication network.

B. Contribution

This paper proposes a new method that combines the bandwidth reduction of EBC and human speech coding named Event Triggered Adaptive Differential Modulation (ETADM). It can be analyzed in terms of Input to State Stability (ISS) and used for nonlinear systems with a stabilizing feedback. The ISS can be assessed in terms of the network delay, reconstruction error due to ETADM and a disturbance input. This can be used to show the robustness of the proposed method to both delay and packet drops. It is also flexible and can be applied to systems with mixed periodic and event based sampling.

C. Outline of this Paper

Section II introduces the type of networked control system for which the proposed method is applied. This is followed by a stability analysis based upon ISS using the ETADM induced signal reconstruction error, network delay and exogenous disturbance to the system as disturbance inputs in Section III. The main results of Section IV obtain the error bounds when using ETADM for signal reconstruction. In Section V, the system is compared with other event triggering mechanisms found in the literature in terms of operating principle. The paper is concluded with an illustrative example to demonstrate the proof of concept in Section VI.

II. Preliminaries

A. Notations

The symbols $\mathbb{R}$ and $\mathbb{Z}^+$, represent the set of real numbers and positive integers (excluding zero) respectively. The Euclidean and standard L_P norms of a variable $v \in \mathbb{R}^n$ ($n \in \mathbb{Z}^+$) are given by $|v|$ and $||v||_p$ respectively unless stated otherwise. If $v \in \mathbb{R}$, the norm $|v|$ is simply the absolute value.

The letters $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ represent the class of functions as defined in [24] (p. 144), where a class $\mathcal{K}$ function is a continuous, strictly increasing, $\mathbb{R} > 0 \to \mathbb{R} > 0$ function which is zero when the argument is zero. A class $\mathcal{K}_\infty$ is an unbounded class $\mathcal{K}$ function. A class $\mathcal{KL}$ function is a $\mathbb{R}^2 > 0 \to \mathbb{R} > 0$ with a class $\mathcal{K}$ first argument and strictly decreasing second argument. For two functions $f, g \in \mathcal{K}$, $f \circ g(r) := f(g(r))$. The function definitions $sgn(r)$ and $sat(r)$ denote the standard sign and saturation function respectively. The applicable parameters for such functions are defined in the text.

The letter k in square brackets is used for periodic (clocked) discrete time sample unless stated otherwise while t_k denotes the time at which the k^{th} sampling event was triggered for a particular event triggering criteria.

From the definitions of [25] with modifications to the notation for clarity, $|v(\cdot, t_d)| := \max_{s\in[t-t_d,t]} |v(s)|$ and $||v(\cdot, t_d)||_T := \sup_{t\in T} |v(\cdot, t_d)|$ where $T \subset [0, \infty)$.

The operator $\vee$ is used to denote the binary maximum operator such that $a \vee b = a$ if $a > b$. The operator $\wedge$ denotes the binary minimum operator. The overbar ($\bar{v}$) on a variable represents its reconstruction. The ceiling and floor of a floating point value are represented by $\lceil\rceil$ and $\lfloor\rfloor$ respectively.

B. Definitions

Definition 1. *A sampling event is an instance when a continuous input variable v is sampled due to v satisfying a set of event triggering conditions $\mathcal{E}$. Let the sampling operator be H, the sampled output,*

$$v_H = H[v, \mathcal{E}]$$

where v_H is a piecewise constant function with $v_H(t) = v_H(t_{k-1})$ for $t \in (t_{k-1}, t_k)$ where t_k is the time when conditions $\mathcal{E}$ are satisfied.

In the case of a discrete variable, the periodic clocking signal implicitly included in $\mathcal{E}$.

C. Control System Model

A NCS can be modeled in the generic form of Fig. 1, consisting of sensors that take direct measurements of the plant as well as remote sensors. The feedback loop via the controller is completed over the network with a network induced error. A real world example of such a NCS is a ship roll stabilizer (Section VI), where the actuator fins that counter the roll have shaft encoders to directly measure the fin position and the inclination or inertial sensor, which are located away from the rest of the control system.

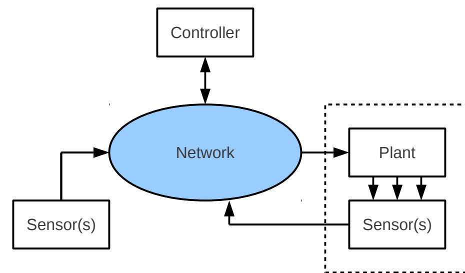

Fig. 1. A Generic Network Control System
The generalized interconnection of the plant, sensors and controller using a communication network.

Consider an autonomous plant with continuous dynamics described by,

$$\dot{x} \quad = \quad f(x, u, w) \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the system state, $u \in \mathbb{R}^{n_u}$ the control input and $w \in \mathbb{R}^{n_w}$ an exogenous disturbance. $n_x, n_u, n_w \in \mathbb{Z}^+$. $f(x, u, w)$ is Lipschitz in x, u and w for $(x, u, w) \in \Omega \subseteq \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_w}$. The origin of (1) is an equilibrium point.

Assumption 1. *There exists a feedback $u = \phi(x)$ capable of stabilizing the plant (1) at the origin when $w = 0$.*

The NCS controller will approximate $\phi(x)$ with the networked induced error e. Hence, the control input $u = \phi(x)+e$.

Assumption 2. *x is measurable.*

Assumption 3. *The system (1) is ISS for inputs w and e.*

Therefore, there exists a C^1 Lyapunov function $V : D \to \mathbb{R} \geq 0$ and $\alpha_1, \alpha_2, \alpha_3, \rho_e, \rho_w \in \mathcal{K}$ such that for a network induced error e,

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \tag{2}$$

$$\dot{V} = \frac{\partial V}{\partial x} f(x, \phi(x) + e, w), \tag{3}$$

$$|x| \quad \geq \quad \rho_e(|e|) \vee \rho_w(|w|) \Rightarrow \dot{V} \leq -\alpha_3(|x|). \tag{4}$$

If V is radially unbounded in $\mathbb{R}^{n_x}$, the system would be globally asymptotically stable and $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$.

D. Speech Coding Techniques

Speech Coding (SC) techniques [26] were developed to reduce bandwidth requirements for human speech. They exploit the high dependence of adjacent samples in human speech. This very property is also found in most practical control systems where the control variables change slowly. Additionally, SC allows large errors in regions where the human ear is insensitive to such errors. A similar principle can be applied to control systems when large errors do not matter for control.

Adaptive Delta Modulation (ADM) [27] is one such technique uses a bi-valued quantization to encode the input signal difference into a single bit. In order to handle large differences, the step size adaptation takes place without feedback between the decoder and encoder. ADM has been used for linear NCS [28] [29] [30]. The reduction of the sample size to a single bit results in significant bandwidth reduction for systems that use Time Division Multiple Access (TDMA) because it enables more data to be compressed into a time slot.

E. Outline of Proposed Method

In the case of packet switched communication networks, transmitting a single bit is inefficient due to the comparatively large packet overhead. Instead the goal in a packet switched network is to reduce the effective packet transmission rate to decrease congestion of the communication network.

In the proposed method (Fig. 2), for each measurable variable ($v[k]$) which can be a state variable or control input, an ADM encoder (Fig. 3) is used to obtain the step size ($S[k]$) instead of the single bit output. The step size is used as the input for event triggering. If the step size exceeds the event triggering threshold, a sampling event is triggered and it is transmitted over the communication network. This effectively reduces the number of packet transmissions compared to a periodically sampled system. According to Definition 1, $\mathcal{E} = \{|S[k]| > e_T\}$ with $z(t) = S[k]$ and zero otherwise for every k. This can be alternatively expressed as

$$z[k] = \begin{cases} S[k] & \text{if } |S[k]| > e_T \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

At the receiving end of the channel, a *lossy integrator* is used to reconstruct the signal ($\bar{v}[k]$).

Assumption 4. *When a sampling event is triggered in a sensor or controller output, the single sampled value is transmitted over a communication network in a packet with the respective destination (i.e. controller input or plant) address.*

This new method will be called Event Triggered Adaptive Differential Modulation (ETADM). The term differential is used instead of delta because the step size is essentially the encoded difference between two adjacent samples.

In this paper, the delayed difference method is used [31] to obtain the difference between two adjacent samples instead of prediction algorithms [32] due to the ability to model the ADM encoder as a discrete time nonlinear system (9). This will be shown in Section IV.

III. Stability Analysis

In this section, the stability of the NCS is established by proving the ISS of the continuous time system (Assumption 3) with respect to the disturbance, network delays and reconstruction error.

The feedback path reconstructs the state $\bar{x}$ and control input $\bar{\phi}$ at a total delay of τ. Therefore, the control system would be given by,

$$\dot{x}(t) = f(x(t), \bar{\phi}(\bar{x}(t-\tau)), w(t)).$$

The network induced error e_{NCS} will consist of an error e_τ due to delay τ (bounded by a maximum time $\tau_{\max}$), an error e_x due to reconstruction of the state vector and an error e_u due to reconstruction of the control inputs. Therefore,

$$\begin{aligned} u &= \phi(x(t)) + e_\tau(t) + e_x(t) + e_u(t), \\ e_\tau(t) &= \phi(x(t-\tau)) - \phi(x(t)), \\ e_x(t) &= \phi(\bar{x}(t-\tau)) - \phi(x(t-\tau)), \\ e_u(t) &= \bar{\phi}(\bar{x}(t-\tau)) - \phi(\bar{x}(t-\tau)). \end{aligned}$$

From property (4),

$$|x| \geq \bar{\rho}_e(|e_\tau| \vee |e_x| \vee |e_u|) \vee \rho_w(|w|) \Rightarrow \dot{V} \leq -\alpha_3(|x|) \quad (6)$$

where $\bar{\rho}_e(r) := \rho_e(3r)$. The stability properties of these errors can be analyzed using the approach of [25] [33] [34].

$$e_\tau(t) = -\int_{t-\tau}^{t} \frac{\partial \phi}{\partial x} f(x(s), \phi(x(s-\tau)) + e_x(s) + e_u(s), w(s)) ds$$

It becomes apparent from the limits that it results in (1) with the feedback delayed by $t_d = 2\tau$. Since the origin is an equilibrium point for (1), by the Integral Mean Value Theorem, $\exists \gamma_x, \gamma_{e_x}, , \gamma_{e_u}, \gamma_w \in \mathcal{K}$ such that e_τ is bounded.

$$\begin{aligned} |e_\tau(t)| \leq\; & \tau(\gamma_x(|x(t,t_d)|) \vee \gamma_{e_x}(|e_x(t,t_d)|) \vee \\ & \gamma_{e_u}(|e_u(t,t_d)|) \vee \gamma_w(|w(t,t_d)|)) \end{aligned} \quad (7)$$

From (6) and (7) the functions $\bar{\gamma}_x, \bar{\gamma}_{e_x}, \bar{\gamma}_{e_u}, \bar{\gamma}_w \in \mathcal{K}$ are defined such that; $\bar{\gamma}_x(r) := \bar{\rho}_e(\tau\gamma_x(r))$, $\bar{\gamma}_{e_x}(r) := \bar{\rho}_e(\tau\gamma_{e_x}(r) \vee r)$, $\bar{\gamma}_{e_u}(r) := \bar{\rho}_e(\tau\gamma_{e_u}(r) \vee r)$ and $\bar{\gamma}_w(r) := \bar{\rho}_e(\tau\gamma_w(r)) \vee \rho_w(r)$. This results in,

$$\begin{aligned} & |x| \geq \bar{\gamma}_x(|x(t,t_d)|) \vee \bar{\gamma}_{e_x}(|e_x(t,t_d)|) \vee \\ & \bar{\gamma}_{e_u}(|e_u(t,t_d)|) \vee \bar{\gamma}_w(|w(t,t_d)|) \Rightarrow \dot{V} \leq -\alpha_3(|x|). \end{aligned} \quad (8)$$

Thus, from (2) and (8) the solution would be,

$$\begin{aligned} |x(t)| \leq\; & \bar{\beta}(|x(t_0)|, t-t_0) \vee \xi_x(||x(\cdot,t_d)||_{[t_0,\infty)}) \vee \\ & \xi_{e_x}(||e_x(\cdot,t_d)||_{[t_0,\infty)}) \vee \xi_{e_u}(||e_u(\cdot,t_d)||_{[t_0,\infty)}) \vee \\ & \xi_w(||w(\cdot,t_d)||_{[t_0,\infty)}) \end{aligned}$$

where $\bar{\beta}(r,t) := \alpha_1^{-1}(\beta(\alpha_2(r,t)))$, $\xi_x(r) := \alpha_1^{-1} \circ \alpha_2 \circ \bar{\gamma}_x(r)$, $\xi_{e_x}(r) := \alpha_1^{-1} \circ \alpha_2 \circ \bar{\gamma}_{e_x}(r)$, $\xi_{e_u}(r) := \alpha_1^{-1} \circ \alpha_2 \circ \bar{\gamma}_{e_u}(r)$ and $\xi_w(r) := \alpha_1^{-1} \circ \alpha_2 \circ \bar{\gamma}_w(r)$. The function $\beta \in \mathcal{KL}$ (from Assumption 3), $\bar{\beta} \in \mathcal{KL}$ and $\xi_x, \xi_{e_x}, \xi_{e_u}, \xi_w \in \mathcal{K}$.

Theorem 1. *For a NCS with feedback that is ISS (i.e. Assumption 3 holds), let the variables x,w,e_x and e_u be bounded such that $|x| \leq \Delta_x$, $|w| \leq \Delta_w$, $|e_x| \leq \Delta_{e_x}$ and $|e_u| \leq \Delta_{e_u}$, let $\delta > 0$ be a small offset, $\Delta > 0$ and the small gain condition hold,*

$$\xi_x(r) < r \textit{ for } \forall r \in (\delta, \Delta)$$

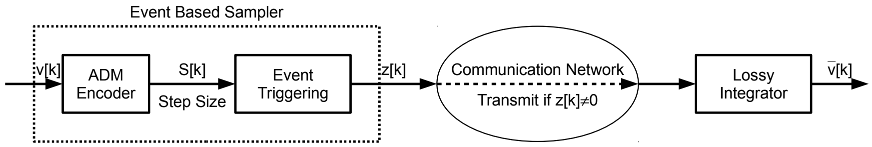

Fig. 2. ETADM Signal Reconstruction of a Sampled Sensor Output (or Control Signal) Transmitted over a Communication Network

for the condition,

$$\delta \vee \bar{\beta}(\Delta_x, 0) \vee \xi_{e_x}(\Delta_{e_x}) \vee \xi_{e_u}(\Delta_{e_u}) \vee \xi_w(\Delta_w) < \Delta.$$

Then the solution of the NCS will be bounded by,

$$\begin{aligned} ||x(\cdot, t_d)||_{[t_0,\infty)} \quad \leq \quad & \delta \vee \bar{\beta}(\Delta_x, 0) \vee \xi_{e_x}(\Delta_{e_x}) \vee \\ & \xi_{e_u}(\Delta_{e_u}) \vee \xi_w(\Delta_w) \end{aligned}$$

and $\exists T > 0$ *such that the solution converges to the ultimate bound*

$$||x(\cdot, t_d)||_{[t_0+T,\infty)} \leq \delta \vee \xi_{e_x}(\Delta_{e_x}) \vee \xi_{e_u}(\Delta_{e_u}) \vee \xi_w(\Delta_w).$$

Proof: The proof is similar to Theorems 1, 2 and 3 of [25] and extending Theorem 1 of [34] to account for network delays, reconstruction noise and disturbances. ■

Theorem 1, establishes the upper bound of the solution of the NCS which depends on the upper bounds of the disturbance and signal reconstruction errors. The upper bound for the signal reconstruction error of ETADM is obtained in the next section.

IV. Main Results

In this section we obtain the main results of this paper. The proposed ETADM scheme (Fig. 2) consists of a sampler to sample each individual state variable x_i ($x = [x_1, x_2, \ldots, x_{n_x}]^T$), an ADM encoder used to obtain the step size (Fig. 3) and the event triggering mechanism. The lossy integrator reconstructs the state variable value $\bar{x}_i$ with an error e_i, where $\bar{x}_i = x_i - e_i$. The same applies for each individual control variable u_i ($u = [u_1, u_2, \ldots, u_{n_u}]^T$).

In the first subsection an upper bound for the signal reconstruction error of ADM is established. Next, we determine the upper bound for the event triggering threshold (e_T) and the error bound for packet drops. Subsequently, the total error bound for ETADM is established which is followed by the error bound for the entire state vector x. Samples of ADM encoded signals are given in [28] (Fig. 5) and [30] (Fig. 4).

A. ADM Error Bound

The ADM decoder (Fig. 4) consists of the step adaptation and a lossy integrator. The sampler fast samples a scalar sensor input at a period T_S and quantizes the signal value. The maximum quantization error is Δ_q.

Assumption 5. *The sampling period T_S satisfies the Nyquist criterion for the sensor input such that the dominant error due to sampling is the quantization error.*

Assumption 6. *All sensors, the controller and the plant have reasonably synchronized clocks for periodic sampling and reconstruction.*

Assumption 7. *The communication network uses packet mode multiple access with a negligible channel access time compared to the periodic sampling rate.*

Assumption 5 is necessary to ensure that there is no significant error in the reconstructed signal due to *aliasing* while Assumptions 6 and 7 are necessary to neglect any effect due to delay variation (jitter). These assumptions ensure that the original continuous time signal can be accurately reconstructed from the sampled one with an error bound of Δ_q (i.e. $|x_i(t) - x_i[k]| \leq \Delta_q$ where $t \in [(k-1)T_S, kT_S)$. Therefore, a discrete time system approach for stability analysis [35] is unnecessary.

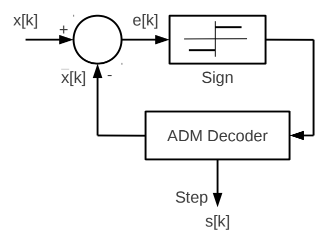

Fig. 3. Schematic of an ADM Encoder

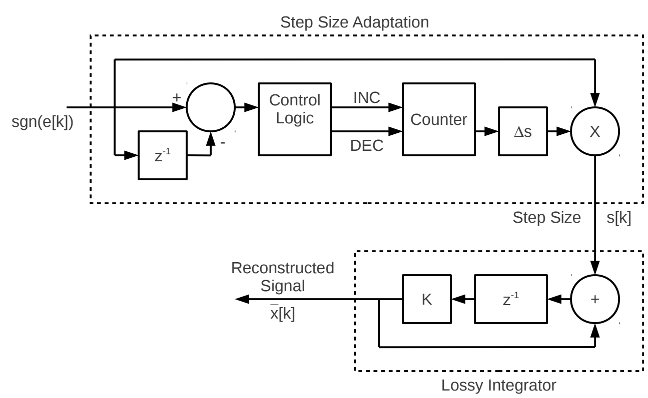

Fig. 4. Schematic of an ADM Decoder

In the step adaptation (Fig. 4), if the input to the control logic is zero, the counter will be incremented. The counter is decremented for a nonzero input. The counter has a strictly positive minimum ($C_{\min}$) and maximum ($C_{\max}$) value beyond which no decrementing or incrementing will take place respectively. This will limit the possible step size to,

$$\begin{aligned} S_{\min} &= \Delta S C_{\min} > 0, \\ S_{\max} &= \Delta S C_{\max} > 0 \end{aligned}$$

where ΔS is the step increment. The step size for the next sample is given by,

$$S[k+1] = E[k] sat(\Delta S E[k] sgn(S[k]) + |S[k]|) \tag{9}$$

where,

$$E[k] = sgn(e_i[k]),$$
$$sat(s) = \begin{cases} s & S_{\min} \le s \le S_{\max} \\ S_{\min} & s < S_{\min} \\ S_{\max} & s > S_{\max} \end{cases}.$$

By the definition of $sgn(s)$, $E[k] = \{-1, 0, 1\}$. It should also be noted that if $E[k_0] = 0$, the value of $S[k+1]$ would become zero $\forall k \ge k_0 + 1$ if $C_{\min} = S_{\min} = 0$.

The transfer function of the lossy integrator is given by,

$$H(z) = \frac{Kz^{-1}}{1 - Kz^{-1}}. \tag{10}$$

For the integrator to be stable, $K < 1$. Unlike a ZOH, the output of a lossy integrator will gradually decay to zero in the absence of an input. For an accurate reconstruction the attenuation of the lossy integrator must be sufficiently high. Hence, the parameter K which determines the attenuation should be near one. When $K = 1$, the integrator is marginally stable.

The reconstructed signal is given by,

$$\bar{x}_i[k] = \sum_{i=1}^{k} K^{k-i+1} S[i] + K^k \bar{x}_i[0]. \tag{11}$$

Henceforth, the asymptotic reconstruction error of ADM is taken as e_{AS}.

Theorem 2. *The asymptotic ADM reconstruction error (e_{AS}) is bounded by;*

$$0 \le |e_{AS}| \le S_{\max}$$

Proof: The possible values of $S[k]$ will be discrete and range from $-S_{\max}, -(S_{\max} + \Delta S), \cdots - S_{\min}$ and $S_{\min}, S_{\min} + \Delta S \ldots S_{\max}$. Each of these values will be due to the state of the counter, hence these discrete values can be taken as individual states. The total number of states will be given by $2(C_{\max} - C_{\min} + 1)$. For convenience the state (i.e. value of $S[k]$) at time k is denoted by the subscript S_k. The variable $E[k]$ which takes values $\{-1, 0, 1\}$ can also be similarly expressed as a discrete state.

Take the positive states of S as set $S+$ and negative states as set $S-$. From (9) if $E_{k-1} = 1$, $S_k \in S+$ and vice versa. If $S_k \in S+$,

$$S_{k+1} = \begin{cases} S_k + \Delta S & E_k = 1 \text{ and } S_k \ne S_{\max} \\ S_{\max} & E_k = 1 \text{ and } S_k = S_{\max} \\ -(S_k - \Delta S) & E_k = -1 \text{ and } S_k \ne S_{\min} \\ -S_{\min} & E_k = -1 \text{ and } S_k = S_{\min} \end{cases} \tag{12}$$

If $S_k \in S-$,

$$S_{k+1} = \begin{cases} -(S_k + \Delta S) & E_k = -1 \text{ and } S_k \ne -S_{\max} \\ -S_{\max} & E_k = -1 \text{ and } S_k = -S_{\max} \\ S_k - \Delta S & E_k = 1 \text{ and } S_k \ne -S_{\min} \\ S_{\min} & E_k = 1 \text{ and } S_k = -S_{\min} \end{cases} \tag{13}$$

When tracking the constant input x_i from an arbitrary initial condition, the system will undergo two types of transients prior to convergence. The first type of transient is when the $\bar{x}_i$ converges towards x. During this transient, the sign of E_k will remain constant ($E_k = E_{k-1} = \pm 1$) and $S_{k+1} = \pm(S_k + \Delta S)$ until the maximum value is reached after which $S_{k+1} = S_k = \pm S_{\max}$.

The second transient will start after $\bar{x}_i$ overshoots x_i. In this stage, the sign of E_k will alternate ($E_k = -E_{k-1}$). The value of S_{k+1} will now gradually decrease and alternate in sign until it converges to a periodic solution. For a periodic solution $S[k] = S[k + n_0]$, the sum of $S[k]$ satisfies

$$\sum_{k=k_0}^{n_0+k_0} S[k] = 0. \tag{14}$$

There are two such periodic solutions[1],

1) when $E_k = -E_{k-1}$ and $|S_k| = S_{\min}$ giving the state transition

$$S_{k+1} = -S_k, \tag{15}$$

2) when $E_k = -E_{k-1} = -E_{k-2} = E_{k-3}$ and $S_{\max} \ge |S_k| \ge S_{\min}$, taking the initial value of S_k as S_0 ($S_{\min} \le |S_k| \le S_{\max}$),

$$\begin{aligned} S_{k-3} &= S_0, \\ S_{k-2} &= -sgn(S_0)[S_0 - sgn(S_0)\Delta S], \\ S_{k-1} &= -sgn(S_0)S_0, \\ S_k &= S_0 - sgn(S_0)\Delta S, \\ S_{k+1} &= S_0. \end{aligned} \tag{16}$$

The first periodic solution will occur if $x_i \ne 0$ and $\exists k_P$ such that the condition,

$$\bar{x}_i[k_P] \in (x_i - S_{\min}, x_i) \cup (x_i, x_i + S_{\min}) \tag{17}$$

is satisfied for the initial conditions S_0 and $\bar{x}[k_0]$. The reconstruction error bound for this cycle is given by,

$$|e_{AS}| = |x_i - \bar{x}_i| \le S_{\min}.$$

The second periodic solution will occur if $x_i \ne 0$ and $\exists k_P$ such that,

$$\bar{x}_i[k_P] \in (x_i - \Delta S, x_i) \cup (x_i, x_i + \Delta S) \tag{18}$$

for the initial value of S_0 and $\bar{x}[k_0]$. The reconstruction error bound for this cycle is given by,

$$|e_{AS}| = |x_i - \bar{x}_i| \le S_0 \le S_{\max}.$$

The lower bound for $|e_{AS}|$ is zero since $sgn(0) = 0$. ■

Theorem 2 provides the asymptotic error bound for ADM for each individual state (or control) variable. In subsequent sections it will be used to obtain the error bound for ETADM for an individual variable followed by the error bound for the entire vector.

Corollary 1. *From (9), should $E_k = 0$, $S[k] = 0$.* i.e. *If $\bar{x}_i = x_i$ it will remain so indefinitely. Since this equilibrium point is unstable, the slightest disturbance will result in the periodic solution (15).*

Claim 1. *Due to the lossy integrator an asymptotic periodic solution will not last. The number of samples a periodic*

[1]From (9) we can exclude the possibility of the encoder converging to a strange attractor.

solution will last ($n_{\max}$) for a given input x_i is upper bounded by,

$$n_{\max} \leq \left\lfloor \frac{\ln\left[1 - \frac{S_p}{|x_i|}\right]}{\ln K} \right\rfloor - 1 \quad (19)$$

where $S_p = S_{\min}$ for periodic solution (15) and $S_p = \Delta S$ for periodic solution (16).

Proof: Since $K \to 1$, from (14) for a periodic solution of period n_P,

$$\bar{x}_i[k+n_p] = K^{n_p+1}\bar{x}_i[k] + \underbrace{\sum_{i=0}^{n_p} K^{i+1}S[i]}_{\approx 0} \approx K^{n_p+1}\bar{x}_i[k]. \quad (20)$$

Thus, the output of the lossy integrator will gradually attenuate. The maximum number of samples ($n_{\max}$) for which condition (17) or (18) will remain satisfied for the respective periodic solution will be given by,

$$|\bar{x}_i[k+n_{\max}]| \approx |x_i[k] - S_p| \approx K^{n_{\max}+1}|x_i[k]|. \quad (21)$$

This gives the upper bound (19) after which it will converge to another periodic solution after a brief transient. ∎

Remark 1. *From the upper bound (19), a periodic solution will only occur for an input range given by,*

$$S_p < x_i < \frac{S_p}{1-K}.$$

When $0 \leq x_i \leq |S_{\min}| \vee |\Delta S|$ (near the origin), the conditions (17) and (18) can still be satisfied. Therefore, periodic solutions can still occur.

If x_i starts from zero (the equilibrium point), from Corollary 1, only the periodic solution (15) will occur. When x_i approaches the origin both periodic solutions (15) and (16) can occur depending on the satisfaction of (17) and (18). However, the lossy integrator will not spontaneously overshoot. Therefore, (21) will never be satisfied and the periodic solution will persist until no reconstruction error occurs ($e_i = 0$).

B. Event Triggered Adaptive Differential Modulation

In ETADM, an event is triggered when the step size exceeds a predetermined threshold. In terms of system functionality, a packet is transmitted only if the step size exceeds the threshold.

Remark 2. *From Theorem 2 for ETADM we can define the set $\mathcal{S} := \{s \in S | s \leq |e_T|\}$. When $s \in \mathcal{S}$, no events will be triggered and in turn no traffic will be generated.*

Remark 3. *From Claim 1, the number of samples for which a periodic solution of type (15) or (16) will remain within $\mathcal{S}$ is upper bounded by (19).*

Claim 2. *In ADM the current cycle of a periodic solution will be sustained for a variable input if the adjacent samples that are highly dependent.*

Proof: Consider a system that has reached a periodic solution for an input x_i. For a periodic solution of type (15), two successive values of the signal reconstruction $\bar{x}_i$ are related by,

$$|\bar{x}_i[k] - \bar{x}_i[k-1]| = KS_{\min} \approx S_{\min}.$$

Taking $y_1 = x_i[k-1]$ and $y_2 = x_i[k]$, the minimum difference between the input and reconstruction is given by $\delta_{\min} = |y_1 - x_i| \wedge |y_1 - x_i| \leq S_{\min}/2$. Let the input change by a small value δ_0 so that the adjacent samples are highly dependent, $|x_i[k] - x_i[k-1]| \leq |\delta_0|$. If $|\delta_0| < \delta_{\min}$, the condition $E_k = -E_{k-1}$ will continue and the current periodic cycle will be sustained.

For periodic solutions of type (16), we consider values that satisfy the condition $|\bar{x}_i[k] - \bar{x}_i[k-2]| = \Delta S$. Taking $y_1 = x_i[k-2]$ and $y_2 = x_i[k]$ results in $|\delta_0| < \delta_{\min} \leq \Delta S/2$. The current periodic cycle will be sustained if $|x_i[k] - x_i[k-3]| \leq |\delta_0| < \delta_{\min}$ and x_i is increasing or decreasing between k and $k-3$. ∎

Therefore, in ADM if the input varies such that adjacent samples are highly dependent, periodic cycles can be sustained. If the periodic solution belongs to $\mathcal{S}$, no traffic will be generated.

Theorem 3. *Signal reconstruction using ETADM is accurate if and only if $\mathcal{S} = \{-S_{\min}, S_{\min}\}$ where $\mathcal{S} := \{s \in S | s \leq |e_T|\}$.*

Proof: Let S_k be the current state of the counter. From (12) and (13) if $S_k = \pm S_{\min}$, $S_{k-1} = -S_k$ or $S_{k-1} = -(S_k + \Delta S)$. Hence the step size is converging or converged ($|S[k]| \leq |S[k-1]|$) to the periodic solutions (15) or $\{(S_{\min} + \Delta S), -S_{\min}, -(S_{\min} + \Delta S), S_{\min}\}$.

For any intermediate state ($S_k \neq \pm S_{\min}$ and $S_k \neq \pm S_{\max}$), the step size can either be converging (or converged) to a periodic solution of the form (15) if $S_{k-1} = -(S_k + \Delta S)$ or increasing if $S_{k-1} = S_k - \Delta S$. If $S_k = \pm S_{\max}$, the step size could only have increased or saturated ($|S[k]| \geq |S[k-1]|$). Also $\pm S_{\max} \notin \mathcal{S}$ or else the lossy integrator will not receive any input.

The lossy integrator cannot resolve the ambiguity if $S_k \neq \pm S_{\min} \in \mathcal{S}$. This will result in a cumulative error of ΔS if the step size is increasing. If $\mathcal{S} = \{-S_{\min}, S_{\min}\}$, the input to the lossy integrator can either be zero in the case of (15) or $\{(S_{\min} + \Delta S), 0, -(S_{\min} + \Delta S), 0\}$ which is periodic. Thus, from (20), the output of the lossy integrator will remain approximately the same. ∎

Theorem 3 states that it is only possible to have $e_T = S_{\min}$. Therefore, in ETADM, if the periodic solution (15) were to occur it will be blocked entirely. The periodic solution $\{(S_{\min}+\Delta S), -S_{\min}, -(S_{\min}+\Delta S), S_{\min}\}$ (16) will be modified to $\{(S_{\min} + \Delta S), 0, -(S_{\min} + \Delta S), 0\}$. The remaining periodic solutions of (16) will be unaffected.

Corollary 2. *From the proof of Theorem 3, if $\mathcal{S} = \{-S_{\min}, S_{\min}\}$, the asymptotic error bounds of ADM and ETADM are equal. Thus, $0 < |e_{ETADM}| < S_{\max}$.*

C. Packet Drop Error Bound

From (11) since $K < 1$, there exists $n < k$ such that for a small positive value $c_0 \in \mathbb{R}$, $c_0 S_{\max}$ becomes negligible. Therefore,

$$K^{k-n+1} \leq c_0. \quad (22)$$

Taking $N := k - n + 1$,

$$\bar{x}_i[k] \approx \sum_{i=k-N+1}^{k} K^{k-i+1}S[i]. \quad (23)$$

Thus, in order to reasonably estimate $\bar{x}_i$, only N previous samples are needed. The effect of all samples prior to $k-N$ on the estimate can be neglected. We shall henceforth call N the number of *significant samples* for c_0. Since x_i is quantized, $c_0 S_{\max} \leq \Delta_q$.

Lemma 1. *The bound on the error due to P consecutive packet drops (e_D) is given by,*

$$|e_D| \leq \frac{(1-K^P)KS_{\max}}{1-K}$$

and the error will persist for only another N samples.

Proof: If the previous P consecutive packets from time k are dropped (23), the signal reconstruction can be expressed as,

$$\begin{aligned}\bar{x}_D[k] &= \sum_{i=k-N+1}^{k-1} K^{k-P+1} S[i], \\ &= \bar{x}[k] - K \sum_{i=0}^{P-1} K^i S[i].\end{aligned}$$

Therefore, the resulting error bound is given by,

$$|e_D| = K \sum_{i=0}^{P-1} K^i S[i] \leq \frac{(1-K^P)KS_{\max}}{1-K}. \quad (24)$$

■

Remark 4. *From (22) after N samples, the error due to a single packet drop $|e_D[k-N]| \leq c_0 S_{\max} \leq \Delta_q$. Hence, the error due to a packet drop at k (and any prior packet drops) will only last for the next N samples. Beyond this, the packet drop will no longer be significant. Hence, the proposed method is robust to a bounded number of consecutive packet drops.*

Remark 5. *The maximum number of consecutive packet drops P can be expressed in terms of the probability of a packet drop $p_D > 0$,*

$$P = \lceil p_D N \rceil$$

If p_D is sufficiently small such that two consecutive packet drops are statistically improbable, $P = 1$.

D. Total Reconstruction Error

For a scaler variable reconstructed using variable, the total reconstruction error for P consecutive packet drops,

$$\begin{aligned}|e_{Total}| &\leq |e_{ETADM}| + |e_D| + \Delta_q \\ &\leq S_{\max} + \frac{(1-K^P)KS_{\max}}{1-K} + \Delta_q. \quad (25)\end{aligned}$$

From (25), taking $\Lambda_i = |e_{ETADM}| + |e_D| + \Delta_q$, it is possible to define a vector of error bounds $\Lambda = [\Lambda_1, \Lambda_2, \ldots \Lambda_i \ldots]^T$. In the case of the control input, the upper bound for the reconstruction error, $|\Delta_{e_u}| = |\Lambda_u|$ where Λ_u is the error bound vector for u.

For the state variables, the upper bound for the reconstruction error is given by,

$$\Delta_{e_x} = \sup_{v \in x} |\phi(v) - \phi(v - \Lambda_x)| \quad (26)$$

where $|\Lambda_x|$ is the error bound vector for x. It becomes apparent that when ϕ is nonlinear (26) becomes an optimization problem.

Remark 6. *From the vector Λ it becomes apparent that it is possible for identical or different ETADM parameters, packet drop statistics and quantization levels to be used for each scalar variable. A notable configuration would be when* mixed sampling *is used. In mixed sampling, some variables would be discretely periodically sampled while others will be reconstructed using ETADM. When mixed sampling is used, the value of $|e_{ETADM}| = 0$ for periodically sampled variables.*

E. Transient Error Bound

Remark 7. *The transient error bound for the system is given by $2x_{\max}$ where $|x_i| \leq x_{\max}$. However, any transient will only last for an approximate maximum time t_{TR} of,*

$$t_{TR} \approx \frac{2x_{\max}T_S}{S_{\max}}. \quad (27)$$

The system will remain stable provided that $2x_{\max}\sqrt{n_x} \in D$.

Remark 8. *From Corollary 2, for ETADM, the asymptotic reconstruction error is upper bounded by $S_{\max}$. Hence, decreasing $S_{\max}$ will reduce the asymptotic reconstruction error. However, from (27), decreasing S_{max} will increase the transient time and generated traffic. Hence, an optimum value for S_{max} exists.*

V. Event Triggering Mechanism Comparison

This section includes a comparison of the proposed method with two existing event triggering methods in terms of reconstruction error bound, traffic reduction and the resulting error and dynamics due to packet drops.

A. Memoryless Event Triggering

1) Reconstruction Error Bound: In MET, the reconstruction error can be given in terms of the input signal.

$$e_R(v) = \begin{cases} v & |v| \leq e_T \\ 0 & \text{elsewhere} \end{cases}$$

It is very similar to a deadband where the error would be e_T outside $|v| \leq e_T$ instead of zero. Thus, for this method of event triggering, the bound on the error is trivially given by, $|e_R| \leq e_T$.

2) Traffic Reduction: When the event triggering variable satisfies the condition $|v| \geq e_T$, periodic sampling has to be used. Therefore, network traffic is reduced only when $|v| \leq e_T$. For ETADM, no traffic is generated when $|v| \leq e_T = S_{\min}$. Also when $|v| \geq e_T$, if $S[k] = S_{\min}$ there will be no network traffic. Therefore, the traffic reduction of MET will be less than ETADM.

3) Packet Drop Error: Consider a variable sampled at T_S being subject to MET. Should a packet get dropped, the next packet deliver the subsequent sample. This will result in an instantaneous error. The system will remain stable provided that the resulting aliasing error (Assumption 5) is within the ISS stability bound.

B. Memory Based Event Triggering

1) Reconstruction Error Bound: For MBET [1] [19] [20] the reconstruction error for a scaler signal is

$$e_R(v) = \begin{cases} v - v(t_k) & |v - v(t_k)| \leq e_T \\ 0 & \text{elsewhere} \end{cases}.$$

where $v(t_k)$ is the stored value of v from the previous event triggered at $t_k < t$. Hence, similar to MET, $|e_R| \leq e_T$.

2) Traffic Reduction: When $|v - v(t_k)| \leq e_T$ no traffic is produced. When an event is triggered when $|v - v(t_k)| \geq e_T$ occurs at t_{k+1}, only a single packet is needed to update the ZOH from $v(t_k)$ to $v(t_{k+1})$ for all possible values of v.

In the case of ETADM if $v[k] > S_{\min} = e_T$ and $v[k] - v[k-1] > S_{\max}$ multiple packets will be needed to overshoot $v[k]$. After the overshoot multiple packets will also be generated during the transients. Hence, the traffic reduction of MBET is likely to be more than ETADM.

3) Packet Drop Error: Should a packet drop occur for event t_k, of a variable sampled using MBET, the system should remain stable for the previously stored value $v(t_{k-1})$ until the next future event is triggered at t_{k+1}. Prediction based upon the measurement of the current state and system dynamics is the basis for self triggering systems [22] [23] [36]. However, to the best of the authors' knowledge, predicting the next event for a nonlinear system using MBET with a random exogenous disturbances is still an open problem. Thus, it is currently not possible to assess the effect of a packet drop on the stability of such a system. The closest result in this direction is the non-passivity of a ZOH during a packet drop when used for robotic teleoperators [**?**].

C. ETADM

This section summarizes the results obtained in this paper for the respective parameters of ETADM for comparison.

1) Reconstruction Error Bound: For asymptotic dynamics, the reconstruction error for ETADM, $e_R \leq S_{\max}$ (from Corollary 2).

2) Traffic Reduction: In ETADM traffic reduction occurs by selective blocking of asymptotic periodic solutions (Theorem 3 and Claims 1 and 2). For transients, ETADM generates the same traffic as ADM.

3) Packet Drop Error: From Lemma 1, ETADM is robust to packet drops because an error that occurs due to a packet drop will only persist for the next N samples. The error bound due to P consecutive packet drops is given by (24).

VI. Illustrative Example

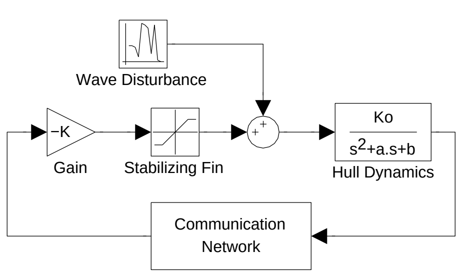

Fig. 5. Nonlinear Ship Roll Stabilizer

Consider the example of a simplified ship roll stabilizer (Fig. 5) which attenuates a bounded wave disturbance (w) on a linear hull using a saturating stabilizing fin. The deflection angle of the hull (x_1) is remotely measured by an inclination sensor, encoded and transmitted along a communication network with a delay. This results in an error e_1. For the system $K, K_0, a, b > 0$ and $x \in \mathbb{R}^2, u \in \mathbb{R}, w \in \mathbb{R}$, the system dynamics (1) are given by,

$$\begin{pmatrix} \dot{x}_2 \\ \dot{x}_1 \end{pmatrix} = \begin{pmatrix} -a & -b \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} (sat(-KK_0(x_1 - e_1)) + w). \quad (28)$$

The origin is the only equilibrium point for (28). Global asymptotic stability for (28) can be shown using the radially unbounded Lyapunov function V.

$$V(x) = \frac{KK_0}{2}(bx_1^2 + x_2^2) + \int_0^{KK_0 x_1} sat(u) du$$

Therefore, from Assumption 3, $\exists \alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and $\exists \rho_w(r), \rho_{e_\tau}(r), \rho_{e_x}(r) := \frac{3|r|}{\theta a} \in \mathcal{K}$ where $\theta \in (0, 1)$ and $e_x = KK_0 e_1$. There is also no restriction on the bound on w, e_X and e_τ. However, realistically $|w| < \pi/2$ and increasing the errors arbitrarily can compromise the disturbance attenuation of the stabilizer.

A. NCS Stability

The system allows the controller to be integrated to the actuator itself. Therefore, the network induced error will only consist of the delay (e_τ) and reconstruction (e_x). The network induced error will affect the control input u_C giving,

$$\begin{aligned} u_C &= -sat(KK_0 x_1) + e_\tau + e_x, \\ u &= -sat(KK_0 x_1) + e_\tau + e_x + w. \end{aligned}$$

For the delay,

$$e_\tau = \int_{t-\tau}^{t} \left(\frac{\partial}{\partial x} u_C \right) f(x, u_C(t - \tau) + e_x, w) dt.$$

The integral which corresponds to a system of delay $t_d = 2\tau$,

$$|e_\tau| \leq \tau KK_0 \left[L|x(t, t_d)| + |w(t, t_d)| + |e_x(t, t_d)| \right]$$

where $L = 1 \vee (|a| + |b|)$ is the Lipschitz constant of f.

$$\begin{aligned} |e_\tau| \leq\ & 3\tau KK_0 L|x(t, t_d)| \vee \\ & 3\tau KK_0 |w(t, t_d)| \vee 3\tau KK_0 |e_x(t, t_d)| \quad (29) \end{aligned}$$

For the NCS, by Theorem 1, the condition for ISS becomes

$$|x| > \frac{3|e_\tau| \vee 3|e_x| \vee 3|w|}{\theta a}. \quad (30)$$

Therefore, the constraint equation for system gain K and delay τ is obtained from (29) and (30), resulting in

$$3\tau KK_0 < 1. \quad (31)$$

TABLE I. SAMPLING METHOD PERFORMANCE COMPARISON ($a = 0.35$)

Sampling	Samples	Reduction (%)	Atten. (dB)
Periodic	100000	(benchmark)	-33.601
MET	96509	3.49	-17.959
MBET	11724	88.28	-12.487
ETADM	20236	79.76	-20.221

B. Numerical Results

Reasonable numerical values are selected for the system. For the slow damping hull dynamics, $K_0 = 4$, $a = 0.35$ and $b = 4$ and for the actuator $|sat(r)| \leq 0.31$. The communication network delay is taken as $4ms$ (4 samples) resulting in a gain (K) of 20 (31). The disturbance affecting the system is bounded with $|\Delta_w| \leq 0.3$. The gain of the lossy integrator ($\bar{K}$) is 0.99. In the ADM encoder, $S_{\min}(= e_T)$, $S_{\max}$ and ΔS are 0.001, 0.0155 and 0.0005 respectively which gives 30 steps. The state variables are sampled at $1kHz$, quantized to 2^{16} levels for $|x_i| \leq 2.5$. These are typical values of commercial analog to digital converting chips.

The given values result in $N = 460$, $|\Delta_q| = 7.625 \times 10^{-5}$, $|e_{ETADM}| \leq 0.0155$ and $|e_D| \leq 0.0153$, assuming a packet drop is a rare event. Therefore, the total reconstruction error,

$$|e_{Total}| \leq |e_{ETADM}| + |e_D| + |\Delta_q| \leq 0.0309.$$

Performance of the ETADM system is checked against a comparable systems using MET and MBET with the same values for e_T. They are subjected to a noisy periodic 0.25 unit amplitude pulsed wave disturbance which results in a hull deflection of approximately 15^o. This value is selected so that in all three systems, the actuator goes into saturation. The pulses had a period of 10s and duty cycle of 50% and were run for 100s. When the performance of each system in terms of traffic reduction is compared with a periodic sampling benchmark (Table I), the traffic reduction of ETADM is greater than MET but less than MBET. It also has the best disturbance attenuation of all three methods. A sample output for the system using ETADM is given in Fig. 6.

Figure 7 gives a closeup of a signal reconstructed using ETADM. The results of Claim 1 and Remark 3 can be observed between 4.9-5s in Fig. 7. Figure 8 shows that the time each individual periodic solution lasts is consistent with the upper bound obtained from (19) for the given input.

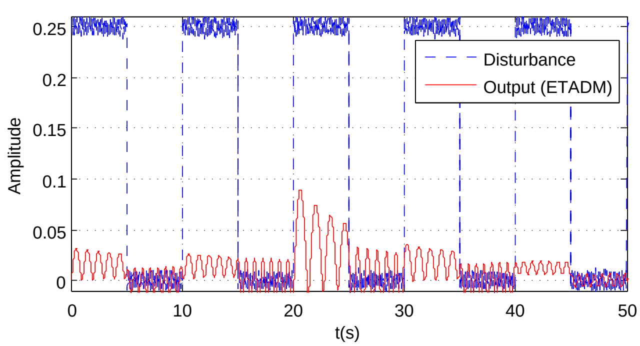

Fig. 6. Sample Disturbance Attenuation of ETADM

It should also be noted that for a highly damped plant ($a = 5$), the traffic reduction of MBET is nearly 20 times better than ETADM with little difference in disturbance attenuation.

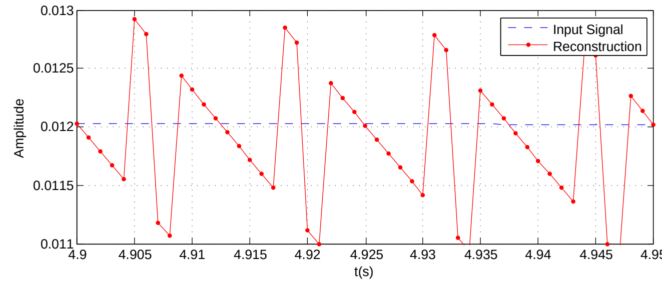

Fig. 7. Sample ETADM Signal Reconstruction

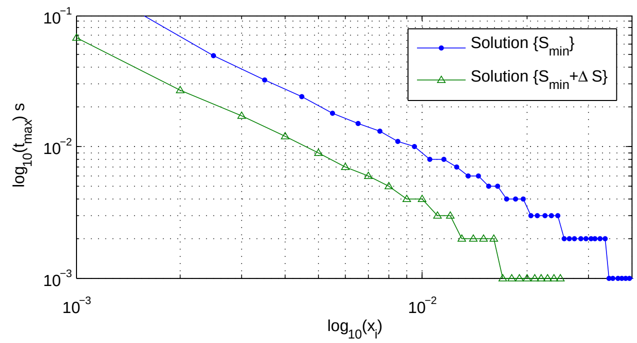

Fig. 8. Upper Bound of the Periodic Solution Duration

Under such circumstances, the main merit of ETADM is its robustness to packet drops.

VII. CONCLUSIONS

This paper proposes a new approach for reducing network traffic in a NCS that combines event triggered sampling and speech coding. It can be used for nonlinear systems with a stabilizing feedback. The stability of such a system can be shown in terms of ISS for the bounds of the exogenous disturbance and network induced error. The network induced error is bounded by the error induced by delay and the bound of ETADM.

In the numerical example for a ship roll stabilizer it significantly reduces network traffic compared to periodic sampling and MET but generates more traffic than MBET (Table I). For low damped plants, despite producing more traffic than MBET, the disturbance attenuation of ETADM higher than both MET and MBET. In addition, ETADM is robust to packet drops. Due to the lack of a formal stability theory for MBET in the presence of a exogenous disturbance, the effect of a packet drop on the dynamics on the system is difficult to assess. In the case of highly damped plants, MBET has a significant advantage over ETADM in terms of traffic reduction. For such plants, the main merit of ETADM is its robustness to packet drops.

TABLE II. SAMPLING METHOD PERFORMANCE COMPARISON ($a = 5$)

Sampling	Samples	Reduction (%)	Atten. (dB)
Periodic	100000	(benchmark)	-36.408
MET	58138	41.86	-26.378
MBET	833	99.17	-26.286
ETADM	18790	81.21	-26.286

The main focus for future work should be on transient analysis of ETADM in order to reduce the transient error for tracking systems. Currently ETADM transmits only a single sample per packet which is can be inefficient for protocols for large packet sizes. Therefore, another necessary future direction is the development of further traffic reduction strategies that exploit the large packet size of modern communication network protocols. In addition it is also necessary to analyze the effects of delay variation on the stability of ETADM.

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